VS 212A Proseminar Optics and Dioptrics of the Eye

Austin Roorda, PhD

Part 1: Light and Spectra





Rodieck, B. The First Steps in Seeing

Relative Spectral Absorptance





Fig. 1(4.3.2). Spectral luminous efficiency functions $V(\lambda)$ and $V'(\lambda)$ defining the standard photopic and standard scotopic observer, respectively. Each function is normalized to have a maximum value of unity which occurs at $\lambda_m = 555$ nm for $V(\lambda)$ and at $\lambda'_m = 507$ nm for $V'(\lambda)$.

figure from Wyszecki and Stiles, 1982



UDT sensors catalog

Part 2: Basic Optics

Rays and Wavefronts







- a) Spherical diverging wavefront formed by a point source
- b) Spherical converging wavefront formed by a lens
- c) Planar wavefront



d) Aberrated wavefront

Vergence

- The vergence of a wavefront is equal to the inverse of the distance from the wavefront to its origin, apparent origin, or destination
- Inverse distance in meters has units of diopters [D]



Distances measured in the direction of the propagating light are given positive values



Diverging wavefront



Distances measured against the direction of the propagating light are given negative values

Vergence of an Object

- When an object is placed in front of an observer, it has a vergence that is equal to 1/(distance from eye to object)
- For example, an object held at 25 cm from the eye has -4 D vergence (ie divergent light)

Refractive Error and Vergence

 Your refractive error is -1X the vergence of the wavefront that emerges from a point source on your retina



Vergence is 4 D, refractive error is -4 D (since a -4 D lens is required to correct the error)

Refractive Error and Vergence

 Your refractive error is -1X the vergence of the wavefront that emerges from a point source on your retina



Vergence is -5 D, refractive error is +5 D (since a +5 D lens is required to correct the error)



n – index of refraction, n=1 (vacuum), $n\sim1.5$ (glass)

The speed of light in any media is...

$$\frac{1}{n} \times c$$

where c = speed of light in a vacuum = 3 x 10⁸ m/s

Application of Snell's Law for Ray Tracing



Light paths through most optical systems can be traced exactly using Snell's Law.

Approximation to Snell's Law

Power series expansion for $sin(\theta)$

$$\sin\left(\theta\right) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$

First order, or paraxial approximation

$$\sin \theta = \theta \implies n\theta = n'\theta'$$

Third order approximation (Seidel)

$$\sin \theta = \theta - \frac{\theta^3}{3!} \qquad \Rightarrow \qquad n \left(\theta - \frac{\theta^3}{3!} \right) = n' \left(\theta' - \frac{\theta'^3}{3!} \right)$$

Approximations

The goal of the approximation is to make a simpler equation that looks like this function first-order third-order fifth-order 2 3 Sin(angle) .3 angle (radians)

Role of Approximations

 The role of the approximation is to generate predictive relationships for ray paths in simple optical systems without using Snell's Law – based ray tracing

 Gauss (1843) used the first order approximation to compute basic object image relationships for spherical refracting surfaces







special case 1: object at infinity => L = 0 & L'=F'

$$F' = \frac{n' - n}{r}$$



special case 2: image at infinity => L' = 0 & L=-F

$$F = \frac{n'-n}{r} = F' \qquad \qquad L' = L + F'$$

Ray tracing



Thick Lenses: Cardinal Points



 $F_E = F_1 + F_2 - \frac{t}{n_2}F_1F_2$

 $F_E = -\frac{n_1}{f_E} = \frac{n_3}{f_E'}$

 $F_V = \frac{-n_1}{f_V}$

 $F_V' = \frac{n_3}{f_V'}$

 $F_{V} = \frac{F_{2}}{1 - \frac{t}{n_{2}}F_{2}} + F_{1} = \frac{F_{E}}{1 - \frac{t}{n_{2}}F_{2}}$

 $F_V' = \frac{F_1}{1 - \frac{t}{n_2}} + F_2 = \frac{F_E}{1 - \frac{t}{n_2}} + F_1$

- $A_1, A_2.... \equiv$ first, second..... surface etc.
- $n_1, n_2, n_3... \equiv$ index of refraction of 1st 2nd 3rd media
- H = primary (or first) principal plane
- H' = secondary principal plane
- P = primary (or first) principal point
- P' = secondary principal point
- N = primary (or first) nodal point *
- N' = secondary nodal point *
- F = primary focal point
- F' = secondary focal point
- $F_1, F_2 \dots \equiv power of 1^{st} 2^{nd} \dots surfaces$
- $F_E = equivalent power$
- $F_v \equiv front vertex power$
- $F'_v \equiv back vertex power$
- $f_E = primary$ equivalent focal length
- f'_{E} = secondary equivalent focal length
- $f_v \equiv first vertex focal length$
- $f'_v \equiv$ back vertex focal length
- I = object distance from first principal plane
- $I' \equiv$ image distance from secondary principal plane
- x = distance from primary focal point to object
- $x' \equiv$ distance from secondary focal point to image

* In this example, the thick lens is in air and the nodal points coincide with the principal points. When the image media is different than the object media ($n_3 \neq n_1$) then they do not coincide. We will discuss this in more detail in the lectures.

$$\overline{A_1 H} = f_V - f_E = n_1 \left(\frac{t}{n_2}\right) \left(\frac{F_2}{F_E}\right)$$
$$\overline{A_2 H'} = f_V' - f_E' = -n_3 \left(\frac{t}{n_2}\right) \left(\frac{F_1}{F_E}\right)$$

Keating convent	ions:
Keating	Class (Freeman)
u,v	l,l'
u _{HI}	1
u ₁	l_v
U,V	L,L'
f _f ,f _b	f_v, f_v
F ₁ ,F ₂	F,F'
H ₁ ,H ₂	Н,Н'
L_1, L_2	A ₁ ,A ₂
neutralizing power	front vertex power

Thick Lenses and Cardinal Points

- Paraxial lens formulas can be used as long as object distance are measured from H and image distances are measured from H' and effective powers are used
- A ray headed toward the first nodal point, emerges from the direction of the second nodal point with the same angle
- Effective power is the inverse of the distance from the principal planes to the focal points. Back and front vertex powers are measured from the lens surfaces

Thick Lenses in Practice (Newport Corp)

	Diameter	EFL		BFL	FFL	T _c	T _{c1}	T_{c2}	T _e	R	R ₂	$R_{_3}$
Model	(mm)	(mm)	f/#	(mm)	(mm)	(mm)	(mm)	(mm)	(mm)	(mm)	(mm)	(mm)
PAC010	6.35	12.7	2.0	10.7	12.6	3.4	2.7	0.7	2.7	8.609	-5.134	-57.155
PAC013	6.35	19	3.0	17.4	18.6	3.2	2.2	1.0	2.6	12.020	-9.145	-30.911
PAC016	6.35	25.4	4.0	23.9	24.8	3.0	2.0	1.0	2.5	15.305	-11.266	-33.538
PAC017	6.35	31.8	5.0	29.8	31.6	3.7	2.7	1.0	3.4	22.359	-13.136	-119.838
PAC018	6.35	38.1	6.0	35.7	37.7	4.5	3.0	1.5	4.2	22.841	-19.467	-74.716
					·							



Bennett and Rabbetts' Schematic Eye



parameters given in Bennett and Rabbetts 3rd ed p 210

Bennett and Rabbetts' Schematic Eye

Radii of curvature		
Cornea	r ₁	+7.80
Crystalline lens: first surface	r ₂	+11.00
Crystalline lens: second surface	r ₃	-6.47515
Axial separations		
Depth of anterior chamber	d ₁	3.60
Thickness of crystalline lens	d ₂	3.70
Depth of vitreous body	d ₃	16.79
Overall axial length		24.09
Mean refractive indices		
Air	n ₁	1
Aqueous humor	n ₂	1.336
Crystalline lens	n ₃	1.422
Vitreous humor	n ₄	1.336
Surface powers		
Cornea	F ₁	+43.08
Crystalline lens: first surface	F ₂	+7.82
Crystalline lens: second surface	F ₃	+13.28
Equivalent powers		
Crystalline lens	FL	+20.83
Eye	Fo	+60.00
Equivalent focal lengths		
Primary (first) PF	Fo	-16.67
Secondary (second) P'F'	f'o	+22.27
Distances from corneal vertex		
First principal point	A ₁ P	+1.51
Second principal point	A ₁ P'	+1.82
First nodal point	A ₁ N	+7.11
Second nodal point	A ₁ N'	+7.42
Entrance pupil	A ₁ E	+3.05
Exit pupil	A ₁ E'	+3.70
First principal focus	A ₁ F	-15.16
Second principal focus	A ₁ F'	+24.09
Refractive state		0

Apertures and Stops

- The *aperture stop* in system is the main light limiting aperture for on-axis objects
- The *entrance pupil* is the aperture stop as seen from object space (imaged through the optics prior to it)
- The *exit pupil* is the aperture stop as seen from image space (imaged through the optics that follow it)
- In the eye the aperture stop is the pupil. The entrance pupil is what you see when you looks at someone's eye (the real pupil is actually a bit smaller and bit further away than it appears)
- The *Chief Ray* is the ray that goes through the center of the entrance pupil (it will also go through the center of the actual pupil). It defines the center of the blur in the presence of defocus.

Entrance Pupil for the Human Eye



L' -327.59

Cornea: radius of curvature: 7.8 mm Pupil: 3.6 mm from corneal vertex Pupil diameter: 6 mm Cornea diameter: 15 mm index inside eye: 1.333

for the pupil

$$l = -.0036 \text{ m} \implies L = \frac{1.333}{-0.0036} = -370.28 \text{ D}$$

 $L' = L + F$
 $L' = -370.28 + 42.69 = -327.59 \text{ D}$
 $l' = -3 \text{ mm}$
lateral magnification is $\frac{L}{-1} = \frac{-370.28}{-370.28} = 1.13$

Reduced Eye Model



Reduced Eye Model

$$L' = L + F$$

By convention, in the human eye

$$K' = K + F_e \implies K = K' - F_e$$

In the reduced eye, k (far point) and k' are measured from the refracting surface.

Whenever k' is equal to the distance from the refracting surface to the retina, then k is equal to the far point of the eye, and K is equal to the refractive error of the eye.

> If, under these conditions, k is at infinity, then the eye is considered to be emmetropic

Part 3: Optics of the Eye

Components of the human optical system



Components of the human optical system



r

.0068

 $= -5.88 \,\mathrm{D}$

Transition from back surface of cornea (n = 1.376) to the aqueous humor (n = 1.336) radius of curvature = 6.8 mm





total power of cornea ~ +43 D
The Pupil is affected by:

Light conditions Attention Emotion Age

Function:

Govern image quality Depth of focus Control light level? The pupil is located to maximize the field of view of the eye





Factors affecting pupil size

- Stimulus Variables
 - light level
 - spectral composition
 - spatial configurations
 - field size
 - spatial structure of field
 - monocular/binocular view
 - accommodative state
 - non-visual stimuli
 - pain
 - noise

- Observer Variables
 - individual differences
 - age
 - day-to-day within observer variance
 - biomechanical factors
 - respiration
 - heart beat
 - cognitive factors
 - arousal, attention, fright
 - workload
 - hedonistic content

Pokorny and Smith, 1997

The range of light intensities in the environment is enormous!



Rodieck, B. The First Steps in Seeing

Crystalline Lens

Gradient index of refraction n = 1.385 at surfaces n = 1.375 at the equator $n \sim = 1.41$ at the center

Little refraction takes place at the surface but instead the light curves as it passes through.

For a homogenous lens to have same power, the overall index would have to be greater than the peak index in the gradient.



total power of lens ~= 21 D

Accommodation

The <u>relaxed eye</u> is under *tension* at the equator from the ciliary body. This keeps the surfaces flat enough so that for a typical eye distant objects focus on the retina.

Accommodation

In the <u>accommodated</u> eye, the ciliary muscle constricts and relaxes the tension on the equator of the lens.

Surface curvature increases.

Power of the lens increases.



Power of the accommodated lens ~= 32.31 D

Optics of the Eye: The Crystalline Lens



courtesy of Adrian Glasser, PhD

Retina:

Images are sampled by millions of rods and cones.

fovea: 5 degrees from optical axis optic disc: 15 deg from fovea, 10 deg from optical axis. —



Cones and Rods are not Evenly Distributed



The retina is a thick-multilayered structure



What is Visual Angle?

It is the angle subtended at the second nodal point by the image It is also equal to the angle subtended at the first nodal point by the object

The nodal points are points in the optical system where the light passing through emerges at the same angle

The second nodal point in the eye is about 16.5 mm from the retina



- 1 radian = 57.29 degrees
- 1 degree = .0174 radians = 17.4 mrad
- 1 minute = .29 mrad
- 1 mrad = 3.44 minutes
- 1 minute = 4.8 microns (depends on axial length)
- 1 foveal cone = 2.5 microns (with intersubject variability)

 letters on an acuity chart are defined by the angles they subtend





1 foveal cone = ~2.5 microns = ~0.5 arcmin

1 deg = ~288 microns



1 foveal cone = ~2.5 microns

= ~0.5 arcmin

20/20 letter = 5 arcmin



ngle

1 foveal cone = ~2.5 microns = ~0.5 arcmin

20/10 letter = 2.5 arcmin



1 foveal cone

- = ~2.5 microns
- = ~0.5 arcmin

Moon = 30 arcmin



Axes and key angles in the Eye

- **Optical axis**: best line joining the centers of curvatures of the optical surfaces
- Visual axis: line from fovea through the nodal points
- Line of sight: line from object through center of entrance pupil that reaches the fovea (chief ray)
- **Pupillary axis**: line from center of curvature of corneal first surface with pupil center
- Angle alpha: angle between optical axis and visual axis
- **Angle kappa**: angle between pupillary axis and visual axis (angle kappa is easily observed as a displacement of the coaxially viewed corneal reflex from the pupil center of a fixating eye)
- Angle lambda: angle between pupillary axis and line of sight

Visual axis and line of sight are often assumed to be parallel, which is only true for distant objects

5 deg

optic disc

10 deg

posterior pole (optical axis)

fovea

Spectral Transmission of the Whole Eye



Boettner and Wolter, 1962

The aging lens



Spectral Transmission of the Lens



Fig. 2(2.4.6). Spectral density curves of the human eye lens determined for the living eye by an objective method. Data for English observers of different ages (ages are shown against the curves; from Said and Weale, 1959). The crosses refer to mean data for two eyes (ages 48 and 53) obtained, after their removal in operations, by a different method (Weale, 1954).

figure from Wyszecki and Stiles, 1982

Absorbing Pigments in the Retina



from Snodderly, IOVS, 1984

Part 4: Image Quality in the Eye

Blur as a function of defocus and pupil size



Blur as a function of defocus and pupil size



Computation of Geometrical Blur Size



 $blur[mrad] = D \times pupilsize[mm]$ $blur[minutes] = 3.44 \times D \times pupilsize[mm]$

where D is the defocus in diopters

Derivation of blur equation



By similar triangles... $\frac{W}{l-x} = \frac{b}{x}$

use small angle approximation to get... $\frac{W}{l-x} = \frac{l\theta}{x}$

$$\theta = \frac{Wx}{l^2 - lx} = W\left(\frac{x - l + l}{l(l - x)}\right) = W\left(\frac{l}{l(l - x)} - \frac{l - x}{l(l - x)}\right) = W\left(\frac{1}{l - x} - \frac{1}{l}\right) = WD$$

where *D* is defocus in diopters

Application of Blur Equation

1 D defocus, 8 mm pupil produces
 27.52 minute blur size ~ 0.5 degrees

Visualize the retinal blur



Hold this cross close enough to your eye until it appears blurry. (about 4 cm). Close your eyelid to form a slit aperture and observe the appearance of the cross. What line appears clear? Why does the other line remain blurry?















Diffraction

"Any deviation of light rays from a rectilinear path which cannot be interpreted as reflection or refraction"

Sommerfeld, ~ 1894

Fraunhofer Diffraction

- Also called *far-field* diffraction
- Occurs when the screen is held far from the aperture.
- Occurs at the focal point of a lens
 (replace s with f').

Fraunhofer Diffraction

rectangular aperture



Sec. 19-3 Rectangular and Circular Apertures 393



square aperture





(d) Figure 19-5 (Continued)




Airy Disk



The Airy Disk

$$\theta = \frac{1.22 \cdot \lambda}{a}$$

 $\theta =$ angle between peak and first minimum (in radians!)

 $\lambda \equiv$ wavelength of the light

 $a \equiv$ pupil diameter

1 radian =
$$\frac{180}{\pi}$$
 degrees
1 degree = 60 minutes of arc
1 minute of arc = 60 seconds of arc

Point Spread Function vs. Pupil Size



Diffraction-limited Eye

Resolution





Resolution

$$\theta_{\min} = \frac{1.22 \cdot \lambda}{a}$$



Diffraction Blur and Snellen Letter Size







AO image of binary star k-Peg on the 3.5-m telescope at the Starfire Optical Range, Albuquerque, NM, September, 1997.

$$\theta_{\min} = \frac{1.22 \cdot \lambda}{a} = \frac{1.22 \cdot 900 \times 10^{-9}}{3.5} = 0.064 \text{ seconds of arc}$$

About 1000 times better than the eve!

Keck telescope: (10 m reflector) *About 4500 times better than the eye!*

Retinal straylight in the human eye

slides courtesy of Tom van den Berg

Thomas J. T. P. van den Berg, Michiel P. J. Hagenouw, and Joris E. Coppens The Ciliary Corona: Physical Model and Simulation of the Fine Needles Radiating from Point Light Sources IOVS, 46: 2627-2632 (2005).

1





Ciliary corona

Actual subjective appearance of straylight: a pattern of very fine streaks, not at all like the circularly uniform (Airy disc-like) scattering pattern of particles of approximate wavelength size



Central diffraction pattern from 2, 3, 4, 50 randomly placed particles

2



Diffraction pattern for 1000 particles, as a function of wavelength, including spectral luminosity effect.





ran den Berg

Straylight (Glare) Equation



where *I* is the retinal illuminance at the distance θ from the glare source of illuminance *E*. *A* is a scaling constant. *n* is usually calculated to be 2.

Equation applies outside of about 1 degree from the glare source. Although 1% at 1 degree seems small, the total flux in the annulus outside of 1 degree can amount to 10% or more.



Aberrations

"Now, it is not too much to say that if an optician wanted to sell me an instrument which had all these defects, I should think myself quite justified in blaming his carelessness in the strongest terms and giving him back his instrument"

Helmholtz (1881) on the eye's optics.



Figure 5-12 Chromatic aberration (exaggerated) for a thin lens, illustrating the effect on the focal length (a) and the lateral and longitudinal misses (b) for red (R) and violet (V) wavelengths.

Chromatic difference of refraction

L' = L + F

by definition, in the human eye

 $K' = K + F_e \implies K = K' - F_e$



where



Longitudinal Chromatic Aberration

Chromatic Difference of Refraction



Fig. 1. Chromatic difference of refraction from three experimental studies2–4 in the visible spectrum and best-fit Cauchy equation (5a), Cornu's equation (5c), and Herzberger's equation to the combined studies. All data were set to be zero at 590 nm. Results of three studies6–8 with measurements in the infrared are also shown; we moved the data from these studies studies to coincide with Eq. (5a) at the lower wavelength (543 nm, Refs. 6 and 7) or at the lowest wavelength (700 nm, Ref. 8). Where shown, error bars indicate standard deviations.



Figure 3. The significance of chromatic defocus depends on luminance. The solid curve shows the luminance spectrum of white-light emitted by the P4 phosphor of cathode ray tubes and arrows mark the amount of defocus if the eye accommodates for 550 nm. When the peak of the luminance spectrum is in focus, most of the light is less than 0.25 D out of focus.

Thibos, Bradley & Zhang, 1989

Transverse Chromatic Aberrartion

Transverse chromatic aberration is a difference in magnification as a function of wavelength. It can be illustrated simply in an aperture lens system.



This also gives rise to Chromatic Difference of Magnification.



Transverse Chromatic Aberration

Centered pupil: TCA=0



Transverse Chromatic Aberration

Decentered pupil



Monochromatic Aberrations: Perfect Eye



Monochromatic Aberrations: Aberrated Eye



Approximation to Snell's Law

Power series expansion for $sin(\theta)$

$$\sin\left(\theta\right) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$

First order, or paraxial approximation

$$\sin \theta = \theta \implies n\theta = n'\theta'$$

Third order approximation (Seidel)

$$\sin \theta = \theta - \frac{\theta^3}{3!} \qquad \Rightarrow \qquad n \left(\theta - \frac{\theta^3}{3!} \right) = n' \left(\theta' - \frac{\theta'^3}{3!} \right)$$



Figure 5-9 (Continued) (c) Astigmatic surfaces in the field of a lens. (d) Use of a stop to artificially "flatten" the field of a lens. The compromise surface between the S and T surfaces is indicated by the dashed line.

Seidel Aberrations : Spherical Aberration



Figure 5-6 Spherical aberration of a lens, producing in (a) different image distances and in (b) different focal lengths, depending on the lens aperture.

Seidel Aberrations : Coma



Figure 5-10 Images of a square grid (a) showing pincushion distortion (b) and barrel distortion (c) due to nonuniform magnifications.

Total Aberrations of the Eye



Fig. 10. Optical linespread functions of the human eye. Each curve represents the normalized distribution of illuminance occurring on the fundus for a thin line source of light. Dots occur at 0.1 min increments. Narrower curve indicates the diffraction image of a line at the given pupil diameter.

Campbell & Gubisch, 1966
Point Spread Function vs. Pupil Size



Observe your own point spread function

•

How do Diffraction and Aberrations Compare with Defocus?

Diffraction-limited eye



How do Diffraction and Aberrations Compare with Defocus?



The Fourier Transform

def: means of transforming a signal defined in the spatial (time) domain to the spatial frequency (frequency) domain

Fourier Transform

- Every time-coded signal can be composed by a superposition of oscillations of different frequencies and phases
- Every 2D pattern can be comprised of a series of sinusoids of different amplitides and phases
- Every object can be considered as being composed of an infinite array of spatial frequencies of all orientations, each with a specific amplitude and phase.

FFT of a heartbeat



Eye Movement Statistics



Power spectrum of eye movements



Fig. 1. Scatter plot of 10,215 eye fixations for the entire visual search experiment. Eye fixations are represented across horizontal (x) and vertical (y) screen coordinates in pixel units. There are clear clusters of fixations in the center and near the boundaries, and intermittent gaps throughout the display.



Fig. 4. Power spectra of vertical eye fixation series from the entire visual search experiment. Total power equals mean squared amplitude. Brown $(1/f^2)$ noise trends emerge. Also shown is a line depicting an exact $1/f^2$ power spectrum.

Fourier Transform

- Every time-coded signal can be composed by a superposition of oscillations of different frequencies and phases
- Every 2D pattern can be comprised of a series of sinusoids of different amplitides and phases
- Every object can be considered as being composed of an infinite array of spatial frequencies of all orientations, each with a specific amplitude and phase.

Frequency Space



- a square wave can be made by adding...
- the fundamental...
- ~~~~ "
- minus 1/3 of the third harmonic
- plus 1/5 of the fifth harmonic...
- minus 1/7th of the 7th harmonic...

© BORES Signal Processing



Fourier Transform

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- Every object can be considered as being composed of an infinite array of spatial frequencies of all orientations, each with a specific amplitude and phase.

FFT of a 2D image



Image

Log (magnitude (FFT))



Power Spectrum of an Image



Power Spectrum of an Image



power spectrum



Yellott's ring (corresponds to the dominant spatial frequency of the photoreceptor array

Power Spectrum of an Image

image



power spectrum



FT of a wavefront

 computes the PSF by adding all the possible interferograms from every pair of points across the wavefront surface.

Application of the FT

- compute the MTF of a wavefront (which spatial frequencies comprise the PSF?)
- compute the PSF from the wavefront (the sum of all the possible interferograms combine to make the PSF)
- compute a convolution

$$f \otimes g = FT\{f\} \times FT\{g\}$$

The PSF is the Fourier Transform (FT) of the pupil function

$$PSF(x_i, y_i) = FT\left\{P(x, y)e^{-i\frac{2\pi}{\lambda}W(x, y)}\right\}$$

MTF Modulation Transfer Function



Michelson Contrast = $\frac{\max - \min}{\max + \min}$

- The modulation transfer function (MTF) indicates the ability of an optical system to reproduce (transfer) various levels of detail (spatial frequencies) from the object to the image.
- Its units are the ratio of image contrast over the object contrast as a function of spatial frequency.
- It is the optical contribution to the contrast sensitivity function (CSF).



2.9 The Modulation Transfer Function

Change in MTF with pupil size



2.9 The Modulation Transfer Function

PSFs for the same eye



Change in MTF with pupil size









Fig. 3. Average MTF's for four pupil diameters (symbols) and the analytical approximations (curves).

Artal and Navarro 1994

Effect of Defocus on the MTF



Spatial frequency



20/10



 $60 \text{ cyc/}_{\text{deg}}$

 $30 \text{ cyc/}_{\text{deg}}$

PTF Phase Transfer Function



Phase Transfer Function



Phase Transfer Function

- Contains information about asymmetry in the PSF
- Contains information about contrast reversals (spurious resolution)

The Importance of Phase



Image quality as a function of pupil size



Part 5: Measurement of the Wave Aberration of the Eye








What is the *Wave Aberration*?



Wave Aberration of a Surface



The Zernike Polynomial

phase = ...

c(1)*sqrt(4)*((1)*r^1)*sin(1*angle) + ... c(2)*sqrt(4)*((1)*r^1)*cos(1*angle) + ... c(3)*sqrt(6)*((1)*r^2)*sin(2*angle) + ... c(4)*sqrt(3)*((2)*r^2+(-1)*r^0) + ... c(5)*sqrt(6)*((1)*r^2)*cos(2*angle) + ... c(6)*sgrt(8)*((1)*r^3)*sin(3*angle) + ... c(7)*sqrt(8)*((3)*r^3+(-2)*r^1)*sin(1*angle) + ... c(8)*sqrt(8)*((3)*r^3+(-2)*r^1)*cos(1*angle) + ... c(9)*sqrt(8)*((1)*r^3)*cos(3*angle) + ... c(10)*sqrt(10)*((1)*r^4)*sin(4*angle) + ... c(11)*sgrt(10)*((4)*r^4+(-3)*r^2)*sin(2*angle) + ... c(12)*sqrt(5)*((6)*r^4+(-6)*r^2+(1)*r^0) + ... c(13)*sqrt(10)*((4)*r^4+(-3)*r^2)*cos(2*angle) + ... c(14)*sqrt(10)*((1)*r^4)*cos(4*angle) + ... c(15)*sqrt(12)*((1)*r^5)*sin(5*angle) + ... c(16)*sgrt(12)*((5)*r^5+(-4)*r^3)*sin(3*angle) + ... c(17)*sgrt(12)*((10)*r^5+(-12)*r^3+(3)*r^1)*sin(1*angle) + ... c(18)*sqrt(12)*((10)*r^5+(-12)*r^3+(3)*r^1)*cos(1*angle) + ... c(19)*sqrt(12)*((5)*r^5+(-4)*r^3)*cos(3*angle) + ... c(20)*sqrt(12)*((1)*r^5)*cos(5*angle) + ... c(21)*sgrt(14)*((1)*r^6)*sin(6*angle) + ... c(22)*sqrt(14)*((6)*r^6+(-5)*r^4)*sin(4*angle) + ... c(23)*sqrt(14)*((15)*r^6+(-20)*r^4+(6)*r^2)*sin(2*angle) + ... c(24)*sqrt(7)*((20)*r^6+(-30)*r^4+(12)*r^2+(-1)*r^0) + ... c(25)*sqrt(14)*((15)*r^6+(-20)*r^4+(6)*r^2)*cos(2*angle) + ... c(26)*sgrt(14)*((6)*r^6+(-5)*r^4)*cos(4*angle) + ... c(27)*sqrt(14)*((1)*r^6)*cos(6*angle) + ... c(28)*sqrt(16)*((1)*r^7)*sin(7*angle) + ... c(29)*sgrt(16)*((7)*r^7+(-6)*r^5)*sin(5*angle) + ... c(30)*sqrt(16)*((21)*r^7+(-30)*r^5+(10)*r^3)*sin(3*angle) + ... c(31)*sqrt(16)*((35)*r^7+(-60)*r^5+(30)*r^3+(-4)*r^1)*sin(1*angle) + ... c(32)*sqrt(16)*((35)* r^7 +(-60)* r^5 +(30)* r^3 +(-4)* r^1)*cos(1*angle)+... c(33)*sqrt(16)*((21)*r^7+(-30)*r^5+(10)*r^3)*cos(3*angle) + ... c(34)*sgrt(16)*((7)*r^7+(-6)*r^5)*cos(5*angle) + ... c(35)*sqrt(16)*((1)*r^7)*cos(7*angle) + ... c(36)*sgrt(18)*((1)*r^8)*sin(8*angle) + ... c(37)*sqrt(18)*((8)*r^8+(-7)*r^6)*sin(6*angle) + ... c(38)*sqrt(18)*((28)*r^8+(-42)*r^6+(15)*r^4)*sin(4*angle) + ... c(39)*sqrt(18)*((56)*r^8+(-105)*r^6+(60)*r^4+(-10)*r^2)*sin(2*angle) + ... c(40)*sqrt(9)*((70)*r^8+(-140)*r^6+(90)*r^4+(-20)*r^2+(1)*r^0) + ... c(41)*sqrt(18)*((56)*r^8+(-105)*r^6+(60)*r^4+(-10)*r^2)*cos(2*angle) + ... c(42)*sqrt(18)*((28)*r^8+(-42)*r^6+(15)*r^4)*cos(4*angle) + ... c(43)*sqrt(18)*((8)*r^8+(-7)*r^6)*cos(6*angle) + ... c(44)*sgrt(18)*((1)*r^8)*cos(8*angle) + ... c(45)*sqrt(20)*((1)*r^9)*sin(9*angle) + ... c(46)*sgrt(20)*((9)*r^9+(-8)*r^7)*sin(7*angle) + ... c(47)*sgrt(20)*((36)*r^9+(-56)*r^7+(21)*r^5)*sin(5*angle) + ... c(48)*sqrt(20)*((84)*r^9+(-168)*r^7+(105)*r^5+(-20)*r^3)*sin(3*angle) + ... c(49)*sqrt(20)*((126)*r^9+(-280)*r^7+(210)*r^5+(-60)*r^3+(5)*r^1)*sin(1*angle) + ... c(50)*sqrt(20)*((126)*r^9+(-280)*r^7+(210)*r^5+(-60)*r^3+(5)*r^1)*cos(1*angle) + ... c(51)*sqrt(20)*((84)* r^9 +(-168)* r^7 +(105)* r^5 +(-20)* r^3)*cos(3*angle) + ... c(52)*sqrt(20)*((36)*r^9+(-56)*r^7+(21)*r^5)*cos(5*angle) + ... c(53)*sqrt(20)*((9)*r^9+(-8)*r^7)*cos(7*angle) + ... c(54)*sgrt(20)*((1)*r^9)*cos(9*angle) + ... c(55)*sqrt(22)*((1)*r^10)*sin(10*angle) + ... c(56)*sgrt(22)*((10)*r^10+(-9)*r^8)*sin(8*angle) + ... c(57)*sqrt(22)*((45)*r^10+(-72)*r^8+(28)*r^6)*sin(6*angle) + ... c(58)*sqrt(22)*((120)*r^10+(-252)*r^8+(168)*r^6+(-35)*r^4)*sin(4*angle) + ... c(59)*sqrt(22)*((210)*r^10+(-504)*r^8+(420)*r^6+(-140)*r^4+(15)*r^2)*sin(2*angle) + ... $c(60) * sqrt(11) * ((252) * r^{10} + (-630) * r^{8} + (560) * r^{6} + (-210) * r^{4} + (30) * r^{2} + (-1) * r^{0}) + \dots$ $c(61) * sqrt(22) * ((210) * r^{10+}(-504) * r^{8+}(420) * r^{6+}(-140) * r^{4+}(15) * r^{2}) * cos(2* angle) + ...$ c(62)*sqrt(22)*((120)*r^10+(-252)*r^8+(168)*r^6+(-35)*r^4)*cos(4*angle) + ... c(63)*sqrt(22)*((45)*r^10+(-72)*r^8+(28)*r^6)*cos(6*angle) + ... c(64)*sgrt(22)*((10)*r^10+(-9)*r^8)*cos(8*angle) + ... c(65)*sgrt(22)*((1)*r^10)*cos(10*angle);



Zernike Polynomials



What are Zernike Polynomials?

- set of basic shapes that are used to fit the wavefront
- analogous to the parabolic x² shape that can be used to fit 2D data

Properties of Zernike Polynomials

- orthogonal
 - terms are not similar in any way, so the weighting of one terms does not depend on whether or not other terms are being fit also
- normalized
 - the RMS wave aberration can be simply calculated as the vector of all or a subset of coefficients
- efficient
 - Zernike shapes are very similar to typical aberrations found in the eye

Relationships Between Wave Aberration, PSF and MTF

The reason we measure the wave aberration



The PSF is the Fourier Transform (FT) of the pupil function

$$PSF(x_i, y_i) = FT\left\{P(x, y)e^{-i\frac{2\pi}{\lambda}W(x, y)}\right\}$$

The MTF is the amplitude component of the FT of the PSF

$$MTF(f_x, f_y) = Amplitude \left[FT\{PSF(x_i, y_i)\} \right]$$

The PTF is the phase component of the FT of the PSF

$$PTF(f_x, f_y) = Phase\left[FT\{PSF(x_i, y_i)\}\right]$$





Point Spread Function



Point Spread Function



Shack-Hartmann Wavefront Sensor



Shack-Hartmann Wavefront Sensor



2.11.4 Principles of the Shack-Hartmann Wavefront Sensor: The Lenslet Array

Shack-Hartmann Images

KW

SM

BD



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Wavefront Maps

(at best focal plane)



Aberrations of an RK patient

Wavefront sensor image



Wavefront aberration



Aberrations of a LASIK patient



Wavefront aberration



Post - RK

Post - LASIK









mт



9.6 8.2 6.9 5.6 4.3 2.9 1.6 0.3 -1.0 -2.3 -5.0

Keratoconus





Contact Lenses for Keratoconus

unaided eye



custom contact lens



PSFs (for 5 mm pupil)

unaided eye



custom contact lens



rms = 4.16strehl ratio = 0.0008 rms = 1.48strehl ratio = 0.004

Metrics to Define Image Quality

Wave Aberration Contour Map



Breakdown of Zernike Terms



Zernike term

Root Mean Square

$$RMS = \sqrt{\frac{1}{A}} \iint \left(W(x, y) - \overline{W(x, y)} \right)^2 dxdy$$

$$A - \text{pupil area}$$

 $W(x, y) - \text{wave aberration}$
 $\overline{W(x, y)} - \text{average wave aberration}$

Root Mean Square



Include the terms for which you want to determine their impact (eg defocus and astigmatism only, third order terms or high order terms etc.)

Point Spread Function



Strehl Ratio

diffraction-limited PSF



PSF through-focus (5 mm pupil)



The highest strehl ratio does not correlate with rms when aberrations are high

Typical Values for Wave Aberration

Strehl Ratio

- Strehl ratios are about 5% for a 5 mm pupil that has been corrected for defocus and astigmatism.
- Strehl ratios for small (~ 1 mm) pupils approach 1, but the image quality is poor due to diffraction.
J CATARACT REFRACT SURG - VOL 32, DECEMBER 2006

Normal-eye Zernike coefficients and root-mean-square wavefront errors

Thomas O. Salmon, OD, PhD, Corina van de Pol, OD, PhD

This metastudy compiles population statistics of over 1300 eyes collected from 10 different labs



For the most part, aberrations in the eye are random. When you average enough eyes together, most terms are no different from zero. The only high order aberrations that is non-zero is spherical aberration, which averages to a small positive value.



A population average of the *magnitude* of the Zernike terms shows that high order aberrations are dominated by 3rd order and spherical aberration.



Like most optical systems, the aberrations diminish as the aperture is reduced.

But unlike turbulence from a telescope, the paraxial regions of the eye have lower aberrations than marginal locations (ie Fried's parameter is not constant)



Overall, the eye's <u>high order</u> aberrations reduce with pupil size. The dashed line indicates the effective diffraction limit, according to Marachel's criterion (RMS < λ /14) for 550 nm light.



Fig. 2. Dynamics of ocular aberrations. (a) Wavefront rms measured at 240 Hz over approximately 4 s. (b) Power spectrum of the signal in (a) showing dynamic behaviour in excess of 30 Hz.

Diaz-Santana et al. Benefit of higher closed-loop bandwidths in ocular adaptive optics, Opt Express, 11: (2003)

 Dynamic Changes in the wave aberrations are caused by

- accommodation
- eye movement
- eye translation
- tear film

Typical Values for Wave Aberration Change in aberrations with age



Monochromatic Aberrations as a Function of Age, from Childhood to Advanced Age *Isabelle Brunette*, ¹ *Juan M. Bueno*, ² *Mireille Parent*, ^{1,3} *Habib Hamam*, ³ *and Pierre Simonet*³

Convolution

Convolution

$PSF(x, y) \otimes O(x, y) = I(x, y)$



Simulated Images

20/20 letters



"I have never experienced any inconvenience from this imperfection, nor did I ever discover it till I made these experiments; and I believe I can examine minute objects with as much accuracy as most of those whose eyes are differently formed"

Thomas Young (1801) on his own aberrations.

Retinal Sampling

Sampling by Foveal Cones

Projected Image



20/20 letter

Sampled Image



5 arc minutes

Sampling by Foveal Cones

Projected Image



Sampled Image



20/5 letter

5 arc minutes

Nyquist Sampling Theorem



Photoreceptor Sampling >> Spatial Frequency



nearly 100% transmitted

Photoreceptor Sampling = 2 x Spatial Frequency



Photoreceptor Sampling = Spatial Frequency



Nyquist theorem: The maximum spatial frequency that can be detected is equal to ½ of the sampling frequency.

foveal cone spacing ~ 120 samples/deg

maximum spatial frequency: 60 cycles/deg (20/10 or 6/3 acuity)



Primate Central Fovea (0.5 deg)





























Appearance of 110 c/deg Interference Fringes



1 deg

DW

RS








Compensation of Ocular Aberrations











Adaptive Optics Flattens the Wave Aberration







AO Improvement in Vision



6.8 mm pupil

AO Improvement in Vision



Adaptive Optics Makes it Possible to See Microscopic Features in the Living Eye

No AO

With AO multiple AO frames



JW right eye 1 deg eccentricity image wavelength = 550 nm

The Trichromatic Cone Mosaic



JW 1 deg nasal



JW 1 deg temporal



AN 1 deg nasal 5 arc min macaque 1.4 deg nasal



