

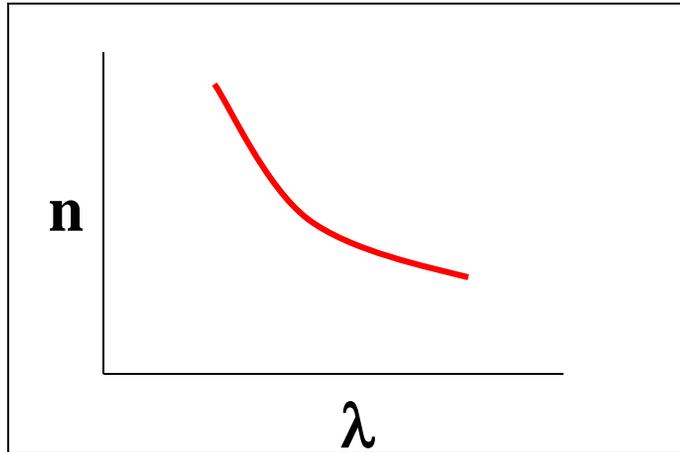
### ***Chromatic and Monochromatic Aberrations***

Aberrations in optical systems can be defined as the failure to get a faithful representation of an object in the image plane. These aberrations take a number of forms. First there are changes in the image with the color. These are referred to as chromatic aberrations. Then there are a subset of aberrations that are called monochromatic aberrations. One type of monochromatic aberration is a failure to get a point image of a point object. Another type of monochromatic aberration is called distortion, which is a failure to get the same shaped image as the object.

### ***Chromatic Aberration (analogous treatment in Keating (20.1-20.5))***

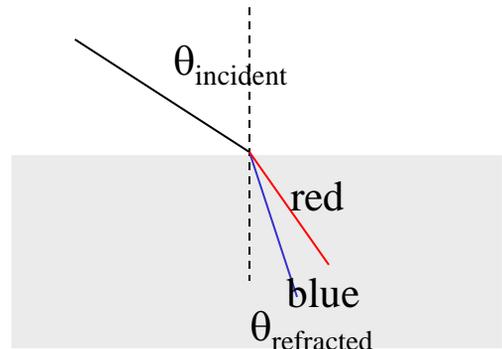
The index of refraction is a number used to describe how fast light travels through media. The speed of light through any media is defined as  $c/n$  where  $c$  is the speed of light in a vacuum and  $n$  is the index of refraction.

Air can be considered a vacuum since it is so rare but technically it is not so and it actually changes with temperature (which gives rise to some optical illusions). The index of refraction of most media changes with the wavelength, some more than others.



The index of refraction is an important factor in determining how light bends at a surface. Recall Snell's Law which states that:

$$n \sin \theta_{\text{incident}} = n' \sin \theta_{\text{refracted}}$$



This change in optical paths as a function of wavelength is called **dispersion**.

Any optical system whose properties are governed in part by the index of refraction (lenses, prism etc) will be affected by dispersion. Different wavelengths will follow different paths.

As an example, the deviation angle in a thin prism is directly affected by the change in index as a function of wavelength, which is the origin of the rainbow effect for light through a prism.

$$d_{\lambda} = (n_{\lambda} - 1)A$$

Where  $d$  is the deviation angle and  $A$  is the apex angle.

The way a lens works is by refracting the light at each point across a curved surface. As the incident beam strikes higher from the optical axis, the angle of incidence gets steeper and the light refracts more so that all rays nearly come to a point. If the blue light gets refracted more because the index is higher, then it follows that the blue focus will be nearer to the lens than the red focus.

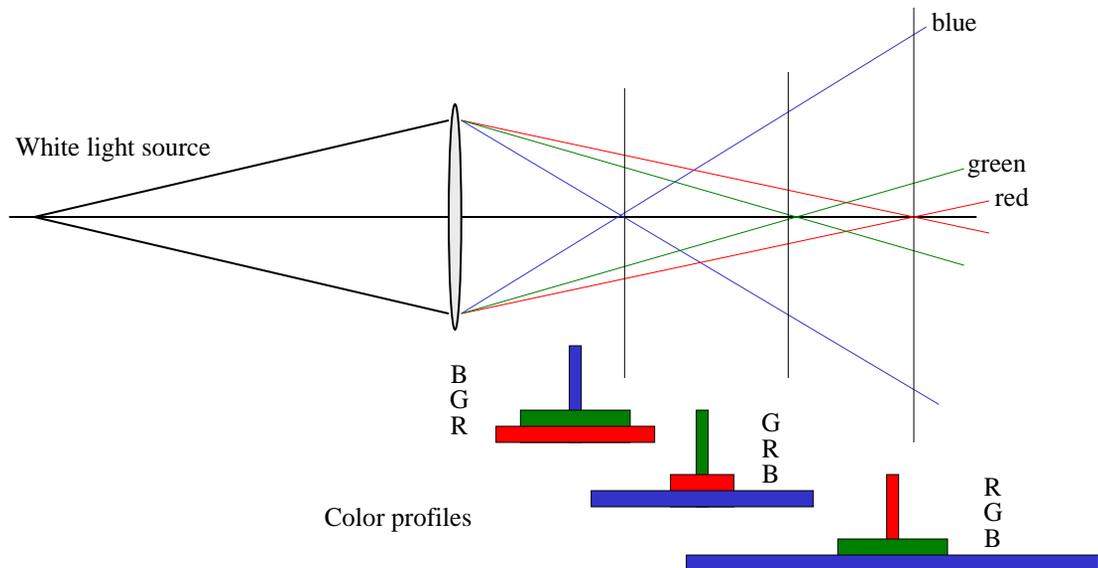
The index of refraction is important in determining the refracting power of a curved surface. Recall that for a spherical surface in air:

$$P = \frac{n-1}{r}$$

The index depends on the wavelength, so the power of the surface depends on the wavelength.

$$P_{\lambda} = \frac{n_{\lambda}-1}{r}$$

The imaging by simple lenses is affected in the following way:

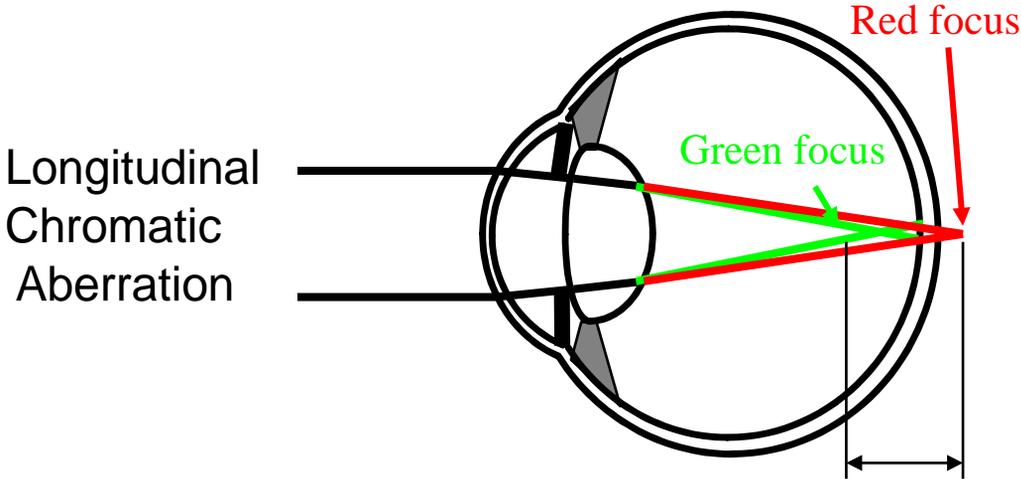


How can we specify the power of a lens, if it varies with the wavelength?

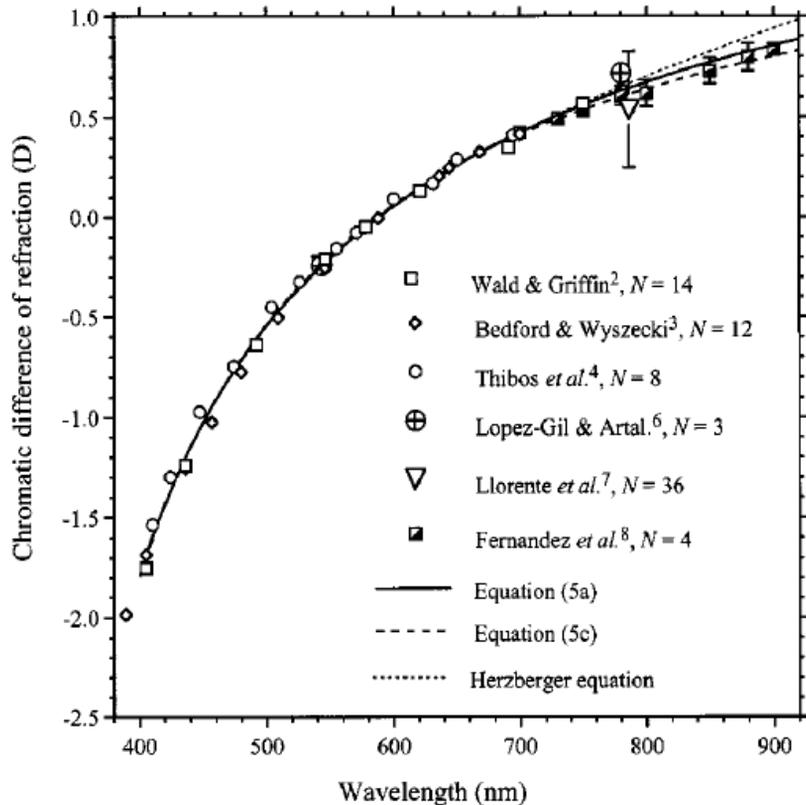
We use the **nominal lens power**, which refers to its power for the D line, or 589 nm light.

***The Red-Green Duochrome Test***

The red-green duochrome test is based on a monocular endpoint in which each eye is tested separately. It is a subjective test that requires responses from the patient and is used to refine the spherical endpoint. Chromatic aberration, the basis of the test, occurs because different wavelengths of light are bent to a different extent. The longer wavelength (red) is refracted less than the shorter (green). If the letters on the red side stand out more, add minus power; if the letters on the green side stand out more, add plus power. Neutrality is reached when the letters on both backgrounds appear equally distinct.

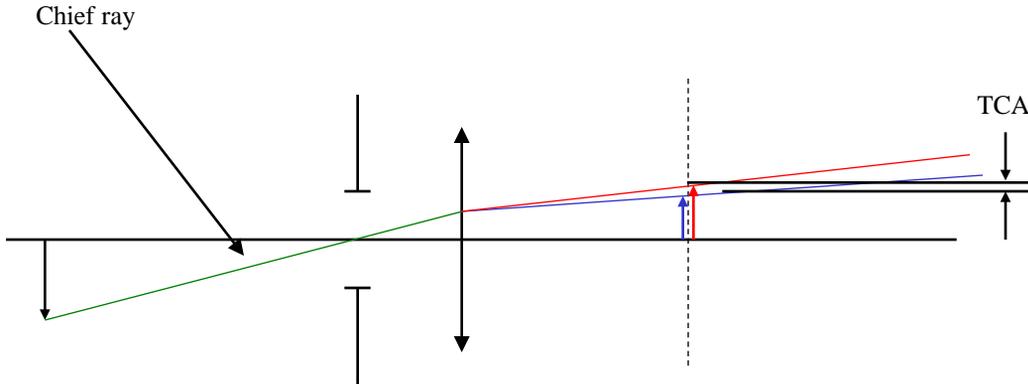


Chromatic  
Difference of  
Refraction

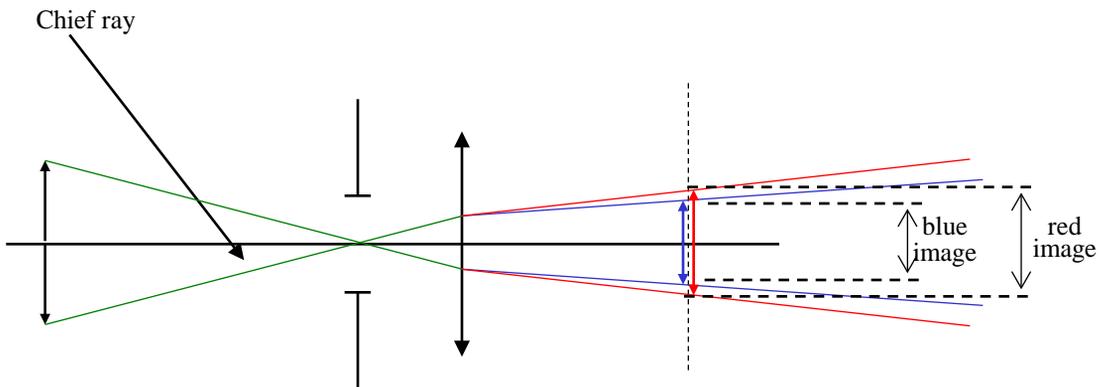


### *Transverse chromatic aberration*

Transverse chromatic aberration is a difference in magnification as a function of wavelength. It can be illustrated simply in an aperture lens system.



This also gives rise to *Chromatic Difference of Magnification*.



To quantify the changes, we adopt the following standard wavelengths between which we can calculate the difference in refraction, or the chromatic aberration. These are:

656 nm	red	C
589 nm	orange-yellow	D
489 nm	aqua-blue	F

Using these conventions, we can classify the chromatic aberration of surface as:  $CA = P_F - P_C$

This is made easier by using a number called the Abbe Number  $\nu$ , a.k.a. refractive efficiency, nu-value, V-value, Abbe number or the constringence.

$$\nu = \frac{n_D - 1}{n_F - n_C}$$

Using this value, we can write:  $CA_{\text{surface}} = \frac{P_D}{\nu}$

Abbe Numbers for common materials:

water	55.6
alcohol	60.6
ophthalmic crown glass	58.6
polycarbonate	30.0
dense flint glass	36.6
Highlite glass	31.0

For thin lenses, where you can assume that the total power is the sum of the power of the two surfaces:

$$CA_{\text{thinlens}} = \frac{P_D}{\nu}$$

If blue focuses in front of red we call it positive chromatic aberration, and negative if red focuses in front of blue.

**Example) Given the following indices of refraction for BK7 glass ( $n_D=1.519$ ,  $n_F=1.522$  and  $n_C=1.514$ ), what is the Abbe Number?**

$$\nu = \frac{n_D - 1}{n_F - n_C} = \frac{1.519 - 1}{1.522 - 1.514} = 64.9$$

**What is the chromatic aberration of a 20 D thin lens made of BK7 glass?**

$$CA = 20/64.87 = .3 \text{ D}$$

### ***The Achromatic Doublet***

There is no way to completely eliminate CA in a lens but there are lenses, called achromats, which are specially designed to minimize chromatic aberration. They are made by combining two lenses. Each lens is made with different media with its own refractive efficiency. For a thin achromat, the total power can be assumed to be the sum of the two thin lens powers.

$$P_{doublet} = P_1 + P_2$$

The total chromatic aberration can be summed in the same way:  $CA_{doublet} = CA_1 + CA_2 = \frac{P_1}{\nu_1} + \frac{P_2}{\nu_2}$

To minimize chromatic aberration, we set CA to zero and solve the equation. This is best done in an example.

Example) Design a 10.00 D achromat using ophthalmic crown glass and dense flint glass.

Solution: The refractive efficiencies for ophthalmic crown glass is 58.6 and for dense flint glass is 36.6. From the condition for zero aberration,

$$0 = \frac{P_1}{58.6} + \frac{P_2}{36.6} \quad (1)$$

We also want to satisfy the condition:  $P_1 + P_2 = 10 \quad (2)$

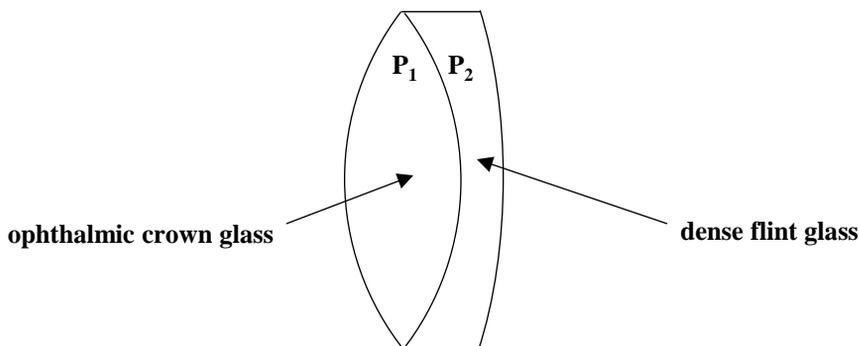
From equation (1):  $P_1 = -\frac{58.6 \cdot P_2}{36.6}$

Sub equation (1) into (2):

$$P_2 \left( -\frac{58.6}{36.6} + 1 \right) = 10 \text{ D} \quad \Rightarrow \quad P_2 = -16.64 \text{ D}$$

Then it follows that:  $P_1 = 26.64 \text{ D}$

The reason this lens is not truly an achromat is because we only satisfied the condition that the C and F focal points are the same.



***The Achromatic Prism***

Similar to an achromatic lens, it is possible to reduce the dispersion in a prism by matching two prisms of different Abbe Numbers.

$$Z = 100 \cdot \tan(d)$$

when  $d$  is small, then

$$Z \approx 100 \cdot (n-1) A$$

where  $A$  is the apex angle

The total power of two thin prism in contact is:

$$Z_{total} = Z_1 + Z_2$$

The total chromatic aberration of the prisms can be summed in the same way:

$$CA_{total} = CA_1 + CA_2 = \frac{Z_1}{\nu_1} + \frac{Z_2}{\nu_2}$$

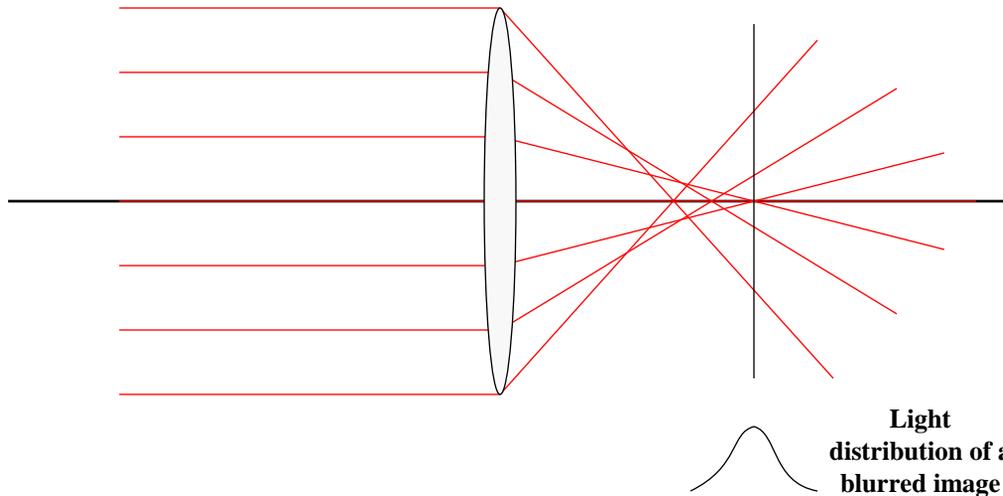
By setting the  $CA_{total}$  to zero, and simultaneously solving both equations, you can design an achromatic prism of any power. See Keating chapter 20, example, 20.2 for an example.

## Monochromatic Aberration

(reference material: Keating 20.9-20.11, 20.15,20.16)

Everything you have learned about object and image formation is an approximation. When light is refracted by spherical surfaces the rays do not all converge to a point, even if they are of one wavelength.

Monochromatic aberrations can arise from surfaces with irregularities (The human eye is a good example) but they also naturally arise from spherical refracting surfaces, or ‘perfect lenses’. If you apply Snell’s law rigorously at every surface for a bunch of rays hitting a lens, you will discover that rays do not all meet at a single point.



**Figure 1:** Schematic demonstrating how monochromatic wave aberration degrades the image quality of a spherical lens.

How can we proceed from the approximation we have learned in the classroom to the real world performance of spherical lenses? A good place to start is with Snell’s law.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

If we want to correctly determine how a lens focuses light, we should apply Snell’s law to every incoming ray and trace out the ray path exactly. This is a very accurate way of describing the image formed by the lens and it fully includes the effects of monochromatic and chromatic aberrations. However, it is very time consuming, and unfortunately this action cannot be condensed into simple mathematical expressions.

Computer ray tracing programs, like ZEMAX, often take this approach. In an effort to make calculations easier, and to aid in our understanding of how lenses work, we make an approximation within Snell’s law for the  $\sin \theta$  term. The approximation, known as the paraxial approximation, substitutes a small portion of the  $\sin$  power series and says it is an adequate description of the  $\sin$  function.

$$\sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$

$$\sin \theta \approx \theta$$

make sure you use units of radians in these equations

**Equation 1: The power series for  $\sin(\theta)$**

**Equation 2: The paraxial approximation, or Gauss’ approximation**

This approximation allows us to derive classic equations like  $V = P + U$ . This approximation is valid for small incident and refracted angles but as you go further from the axis of a lens, or as the object gets bigger, this approximation does not hold anymore. Equations based on the Gauss approximation do not have the ability to describe aberrations. What is needed is a middle ground between the full-blown Snell’s law approach and the oversimplification of the paraxial formula; one that offers a faithful representation of the image, but also eases the burden of calculation. This approximation is found in the **Seidel approximation**.

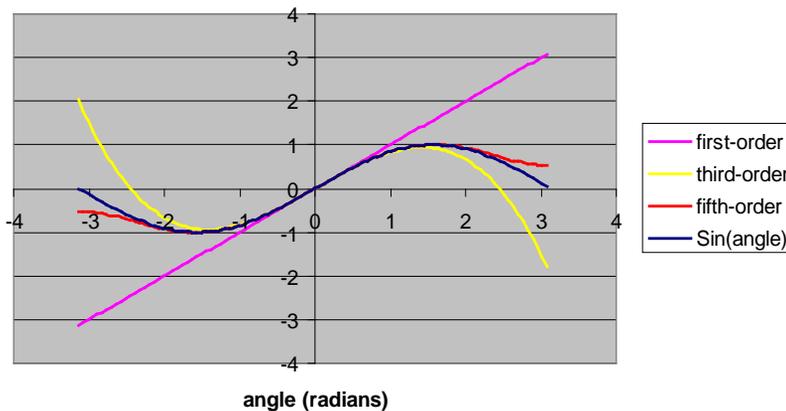
The paraxial formula told us that the light emerging from a lens had a spherical wavefront. Seidel found that the wavefront emerging from a lens was close to spherical but it had some very important and predictable changes. The deviation from an ideal spherical wavefront or **reference sphere** is called the **wave aberration**.

In order to explain these aberrations, Seidel added one more step to the complexity to the approximation of sin in the year after Gauss' death.

$$\sin \theta = \theta - \frac{\theta^3}{3!}$$

**Equation 3: The Seidel approximation**

Using this more complicated approximation allows us to describe rays that make a large angle with the refracting surface. This is because the approximation more closely approximates the real sin function, as can be seen in Figure 2. The blue line is  $\sin \theta$ , the pink line is the paraxial approximation and the yellow line is the Seidel approximation.

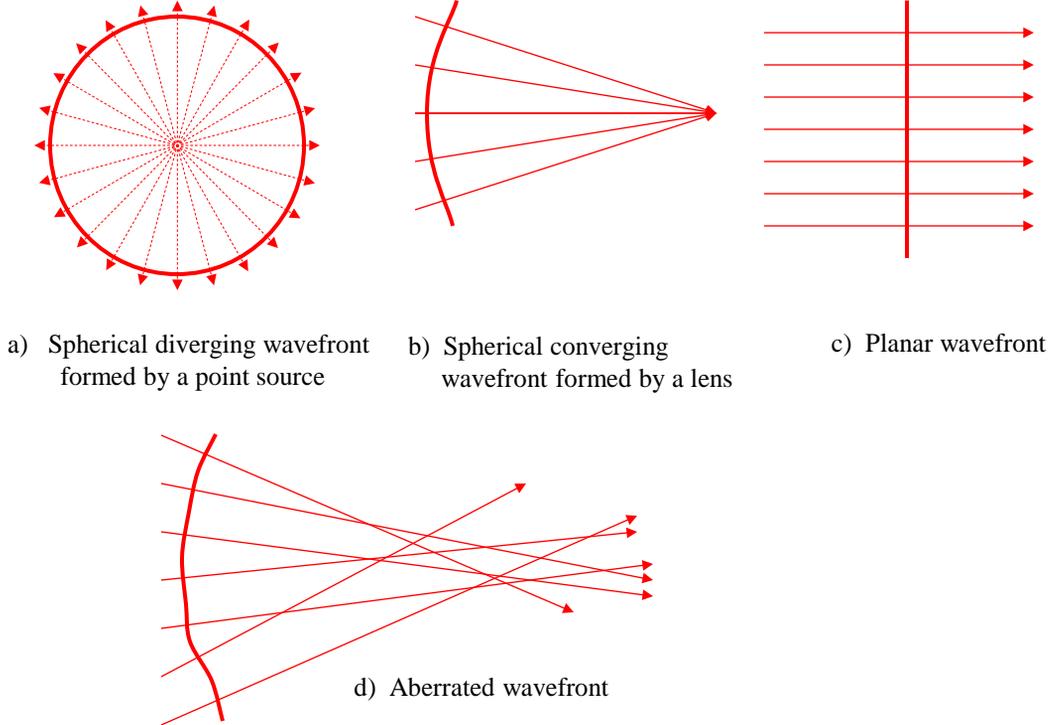


**Figure 2:** graph showing that adding additional terms of the sin power series to the approximation results in a more accurate approximation of sin.

Employing the Seidel approximation allows us to calculate some of the most dominant aberrations. It also sets the stage for the wavefront aberration. First, let's define the term wavefront.

### *The Wavefront*

Like rays, the wavefront is another way to describe how light travels. The wavefront is a line that joins all the photons that emerge from a common source at any given instant in time. Or, if you think of light as a wave (like ripples when you drop a stone into a puddle), the wavefront is a line or surface that follows the crest of any wave. For example, a point source emits light in all directions. The rays radiate outward, and the wavefront is spherical. If all rays after emerging from a lens converge to a point, the wavefront is a section of a sphere. If a beam of light is travelling parallel, the wavefront is a flat plane. The wavefront is useful because it is a single line whose shape can be defined mathematically. This is diagrammed in Figure 3 below.



**Figure 3:** The wavefront is a line that joins all the photons that emerge from a common source at any given instant in time. Here we see **a)** spherical wavefront formed by light exiting a point source **b)** rays converging to a point after emerging from a lens **c)** planar wavefront **d)** aberrated wavefront.

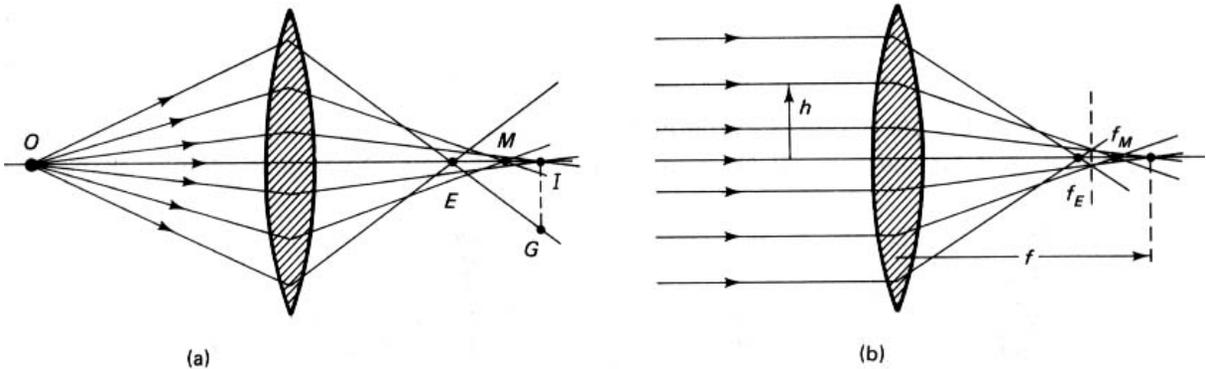
Seidel's calculation showed the wavefront error could be thought of as five separate monochromatic aberrations as well as chromatic aberration (the discussion of chromatic aberration is omitted here and referred to an earlier lecture). The five monochromatic Seidel aberrations can be broken into two groups.

spherical aberration	}	<b>Aberrations that cause image blur: these aberrations are deviations from spherical converging wavefronts. These aberrations result in blur.</b>
coma		
astigmatism		
curvature of field	}	<b>Aberrations that cause image dislocation: these aberrations do not cause deviation from a spherical converging wavefront, but the wavefront is moved from the ideal position, causing a crisp image to be formed in the wrong location.</b>
distortion		

Let's discuss each Seidel aberration term

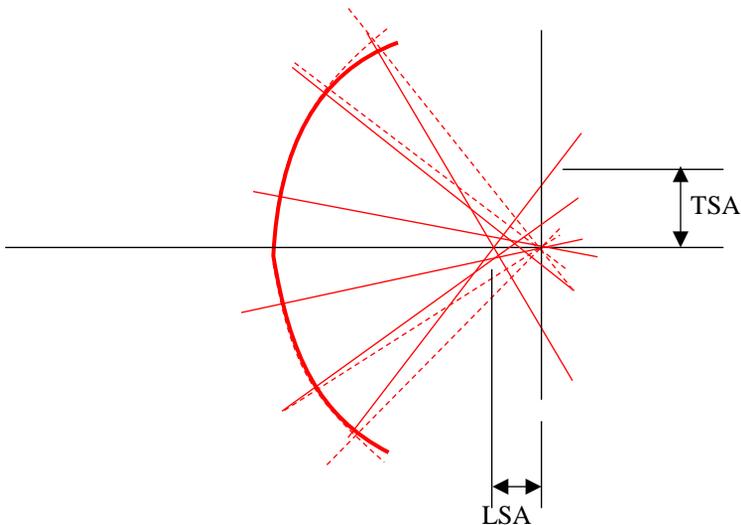
## Spherical Aberration

This aberration affects images of objects on and off the axis. It is an important aberration to consider when designing spectacle lenses, microscopes or other lens systems. It is the **only** Seidel aberration that occurs for light sources that are on axis, but will also occur for light sources that are off axis.



**Figure 4** Spherical aberration of a lens, producing in (a) different image distances and in (b) different focal lengths, depending on the lens aperture.

Rays that are refracted further from the axis get refracted more (or less) than for a spherical wave. In other words, the focal pt depends on the height from the axis that the ray hits the lens.



When the marginal rays converge more than the paraxial, it is called positive spherical aberration, and when the marginal rays converge less than the paraxial, it is called negative spherical aberration.

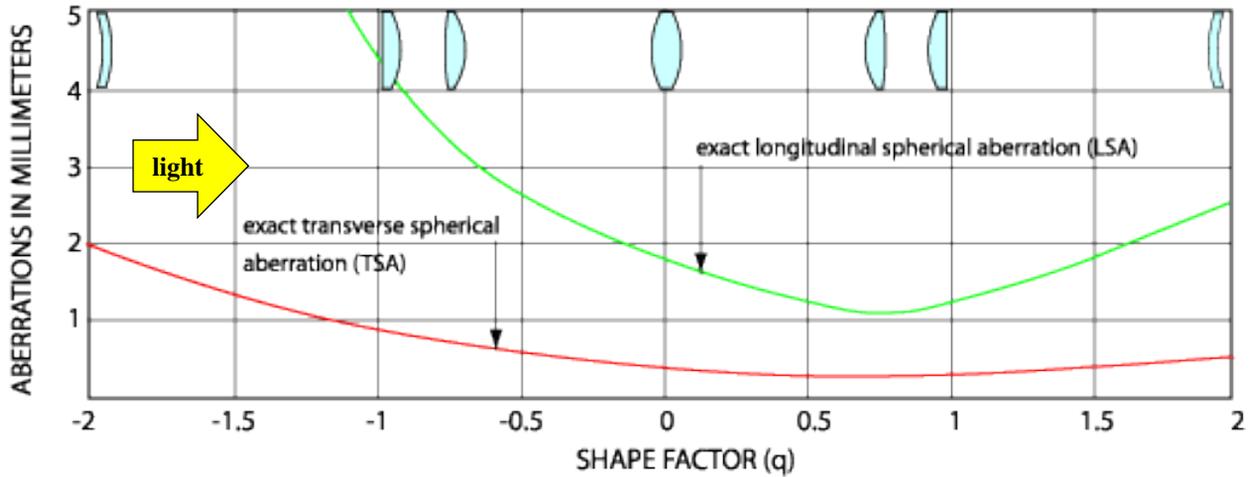
$$LSA[D] = P'_{\text{marginal}} - P'_{\text{paraxial}}$$

or...

$$LSA[m] = f'_{\text{paraxial}} - f'_{\text{marginal}}$$

**Equation 4:** Mathematical description of longitudinal spherical aberration in diopters and in distance.

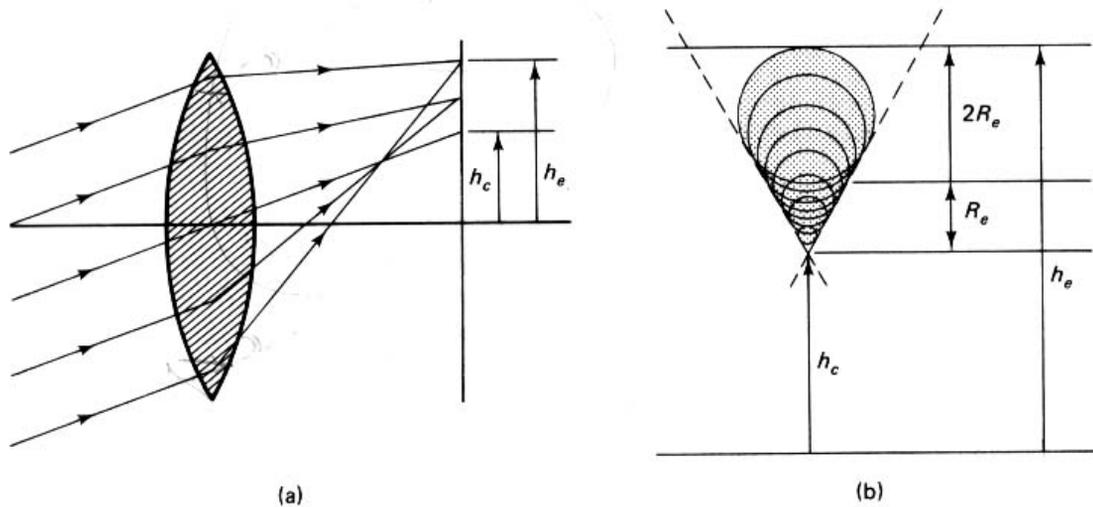
The shape of the lens is a critical factor that governs the amount of spherical aberrations. All these lenses have the same focal length, and they all have positive spherical aberration, but of very different amounts. In the figures below, the light is coming in from the left.



A meniscus lens with the concave side facing the parallel beam produces the most spherical aberration.

## Coma

This aberration affects off axis objects.

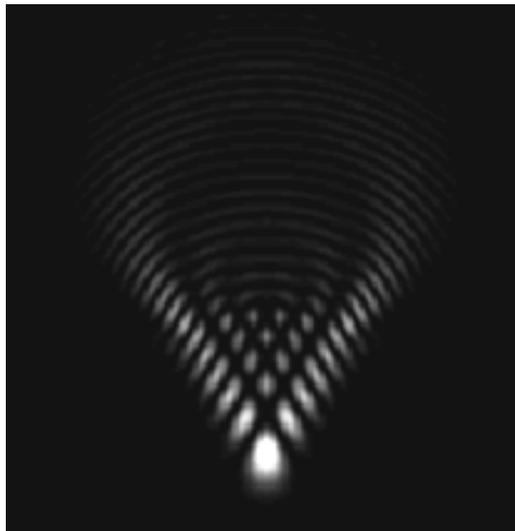


**Figure 5:** a) cross-sectional diagram of the deviation of rays associated with coma. B) comet-shaped blur associated with coma.

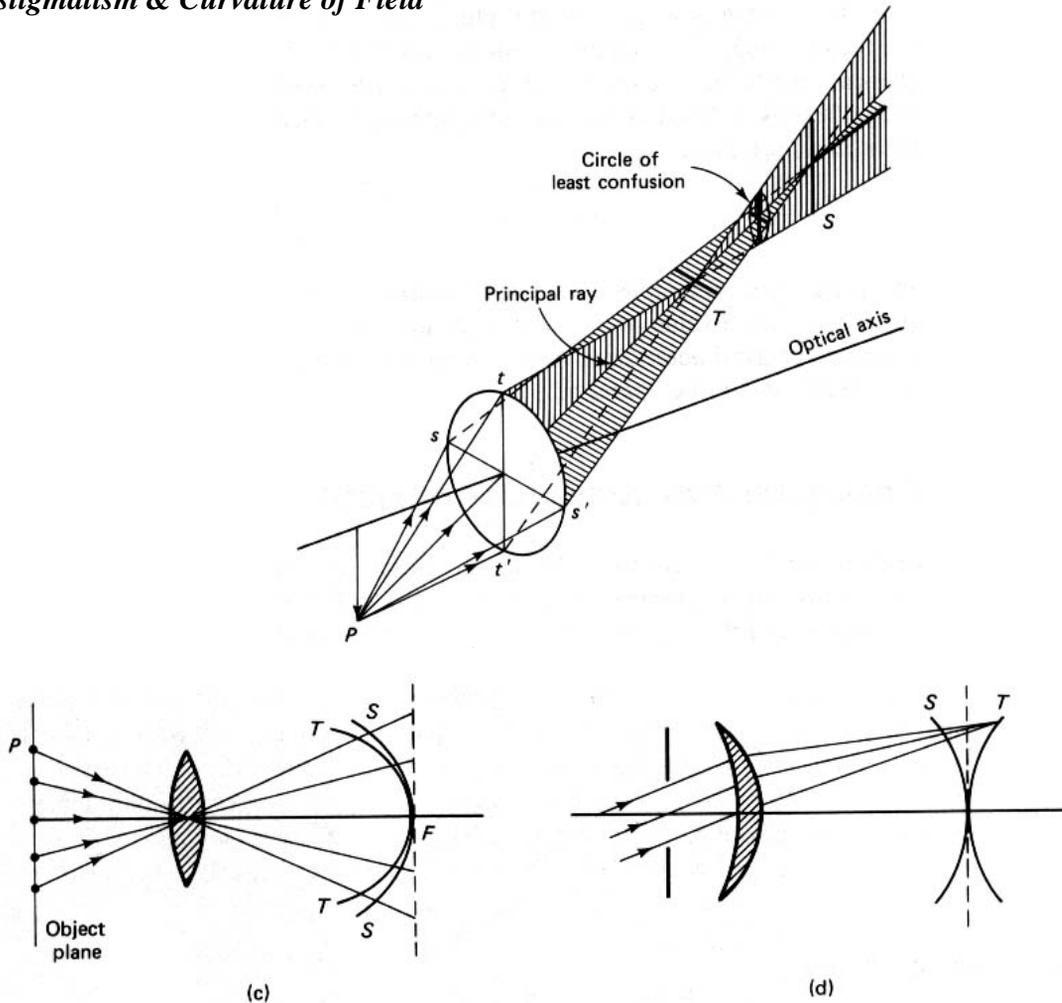
You can think of this aberration in terms of the prism deviation. The prism deviation changes nonlinearly with incident angle. When the beam comes in off-axis, the prism effect of the lens is different at the two corresponding points on either side of the axis. As a result, rays are refracted differently on one side than the other. The result is a comet shaped blur, hence the name coma.

Coma does not exist on-axis in centered optical systems with spherical surfaces. However, since the eye has irregular surfaces, it often has coma along the axis. So rather than seeing a point, you might perceive a comet-shaped blur on your retina.

**Figure 6:** This figure shows the PSF for vertical coma.



## Astigmatism & Curvature of Field



**Figure 7** (Continued) (c) Astigmatic surfaces in the field of a lens. (d) Use of a stop to artificially "flatten" the field of a lens. The compromise surface between the S and T surfaces is indicated by the dashed line.

### Radial, or Off-Axis Astigmatism

Astigmatism arises from toric surfaces but it also occurs when you image off-axis objects. The light that hits the lens sees it differently along the two different axes resulting in two focal points.

- The tangential plane is the plane containing the optical axis and the chief ray from the object. The tangential axis is a line on the lens that is in the tangential plane. (*recall that the **Chief Ray** is the ray that passes through the center of the entrance pupil of the optical system*)
- The sagittal plane is perpendicular to the tangential plane and contains the Chief Ray.

The tangential axis appears to have greater curvature and hence focuses the light earlier to form a horizontal line focus. The sagittal axis appears to have less curvature and causes the vertical line focus that is further away.

The difference in focal distances gives rise to a phenomena called curvature of field where the best focal point with off axis objects follows a **curved surface**. In a simple lens, the tangential focal plane is steeper and more curved than the sagittal plane. Certain lens designs, like that shows in 'd' above, reduce this problem. The eye reduces this problem by curving its image plane, or the retina.

For thin lenses, radial astigmatism can be computed as:

$$P_t = P \left[ \frac{2n + \sin^2 \phi}{2n \cos^2 \phi} \right]$$

$$P_s = P \left[ 1 + \left( \frac{\sin^2 \phi}{2n} \right) \right]$$

When  $n=1.5$  and the angle is less than 20 degrees.

$$P_t = P \left[ 1 + \frac{4\phi^2}{3} \right]$$

$$P_s = P \left[ 1 + \frac{\phi^2}{3} \right]$$

$$RA = P_t - P_s = P\phi^2$$

Make sure that  $\phi$  is in radians whenever you use the thin lens approximation

Why is this important? Keating has some good examples.....

Radial astigmatism is generated by looking obliquely through a spectacle lens. For people with significant refractive errors, this can affect the correction when they do not look through the optical axis. When you look off axis, the astigmatism of the lens increases.

The average power also changes, since the equivalent sphere is the average of the tangential and sagittal powers, and both go up as the angle increases.

This causes a problem when high myopes read through the lower part of their lenses. A partial solution is to tilt the lens so that the error is shared between distant and near viewing. This is referred to as the pantoscopic tilt.

### Example

What is the change in astigmatism when you look through a 5D lens ( $n=1.5$ ) tilted at a downward angle of 15 degrees?

Solution:

15 degrees converts to 0.262 radians

$$P_t = 5 \left[ 1 + \frac{4(0.262)^2}{3} \right] = 5.46D$$

$$P_s = 5 \left[ 1 + \frac{(0.262)^2}{3} \right] = 5.11D$$

So the radial astigmatism is 0.35 D and the equivalent spherical correction of the tilted lens is: 5.28 D

Which axis has the higher power? The tangential axis has the higher power. The tangential plane contains the chief ray and the optical axis of the lens, so the tangential axis in this case is the vertical axis.

(see similar examples in Keating)

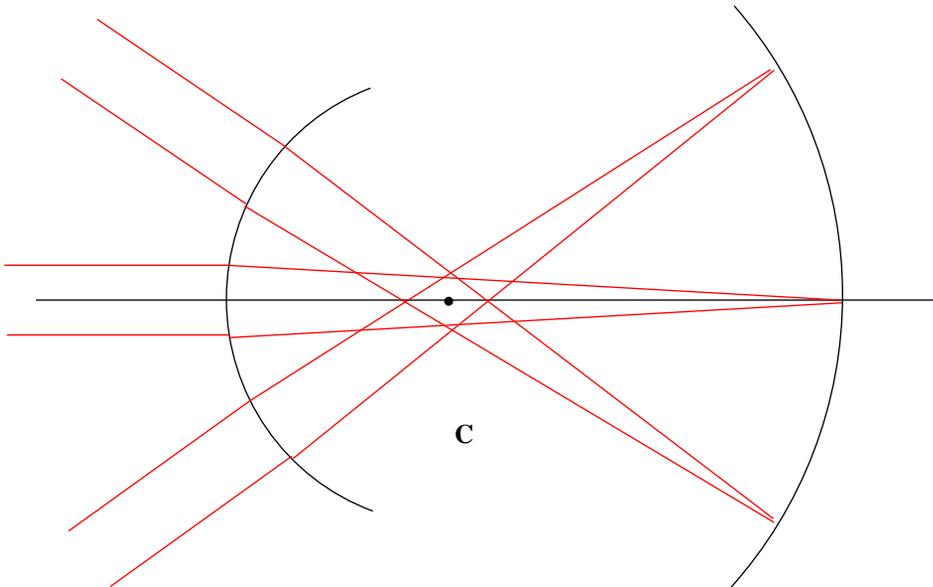
### *Curvature of Field*

Curvature of field can occur without radial astigmatism. Take, for example a single refracting surface with an index of refraction of 1.5 and a radius of curvature of 10 cm.

The power of the surface is +5 D

Parallel rays focus at a distance of.  $f' = n/P = 1.5/5 = 0.3m$

no matter what the angle. So it follows that the image plane must follow a curved surface.



The curvature of the image surface is given by :  $K = P/n$

Where P is the power of the surface and n is the index of refraction of the material into which the light is refracting. For multiple surfaces, the curvature of field is:

$$K = \sum_j \frac{P_j}{n_j}$$

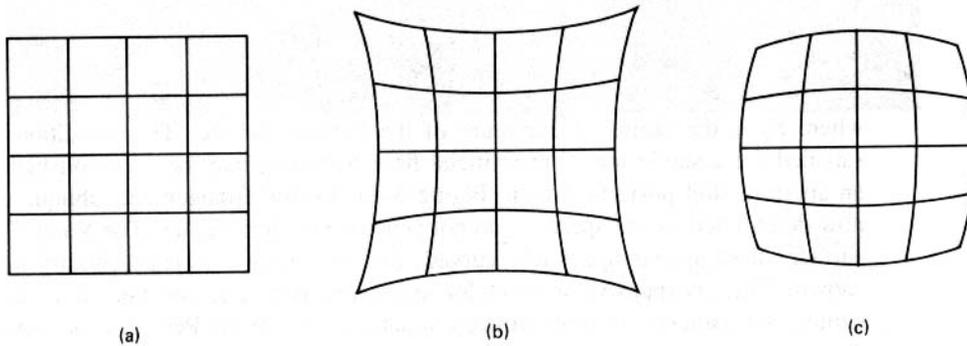
This is called **Petzval's equation**

Petzval's equation also holds for thin lenses.

Petzval's condition describes an optical configuration where Petzval Curvature is equal to zero. I will show an example of this in class.

## Distortion

This aberration affects the way an object is mapped into image space (or a change with magnification with height off-axis).



**Figure 8** **Figure 5-10** Images of a square grid (a) showing pincushion distortion (b) and barrel distortion (c) due to nonuniform magnifications.

In some lens systems, magnification increases as you go further off-axis. In that case, you get pincushion distortion. Corners are more magnified so they push out further than the sides.

Whenever the magnification becomes less as you go off-axis, the corners get pushed in and you see a barrel distortion.

Skip Keating sections on lens design, spherocylindrical lenses and mirrors.

***Demo #1: Aberrations in normal lenses***

Use a laser pointer and an objective lens to generate a point source.

The laser generates a parallel beam.

The objective has a power of

$$P_o = -\frac{M}{g} = -\frac{-40}{0.16} = 250 \text{ D}$$

Sketch the system.

Use a high power lens to demonstrate that light from a point source does not converge to a point in the image plane.

Use another lens to show the same. (Quality depends on the lens)

Use a pair of trial lens to show that coma dominates for large pupils.

Add an aperture to show the astigmatism dominates for small pupils.

***Demo #2: Aberrations in the eye***

Discuss how aberrations are formed on the retina.

Discuss how aberrations are perceived (inverted)

Use the small red LED or video display to display a small object.

Dim lights.

Ask students to block the aberrated parts of their pupil.

Determine how many students have positive spherical aberration.